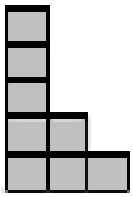
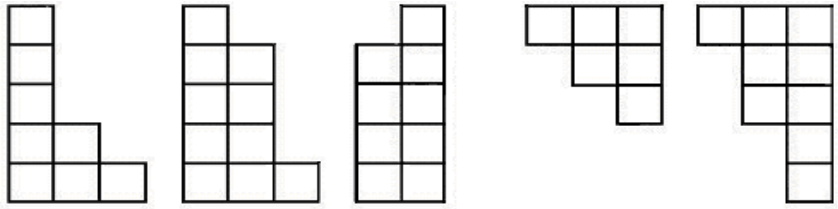
International Contest-Game MATH KANGAROO
Canada, 2007

Grade 5 and 6
SOLUTIONS

Part A: Each correct answer is worth 3 points.

1. Which of the five pieces below fits together with the one given on the right to form a rectangle?





A) B) C) D) E)

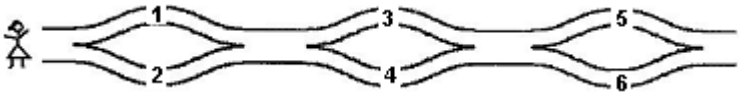
Solution: The piece completing the given figure has to contain a L-shape with two adjacent rows of 4 and 3 squares (to fit the part shown).
Answer: B.

2. It takes Kanga the Kangaroo 6 seconds for every 4 jumps. How long does it take her to do 10 jumps?

A) 10 sec. B) 12 sec. C) 15 sec. D) 18 sec. E) 20 sec.

Solution: Since it takes 6 seconds for every 4 jumps, it will take 3 seconds for every 2 jumps. Therefore, it will take (10/2)×3=15 seconds to do 10 jumps.
Answer: C

3. Zita walked from the left to the right and wrote the numbers she saw along the roads in her notepad. Which of the following groups of numbers could be the numbers written by Zita?



A) 1,2 and 4 B) 2,3 and 4 C) 2,3 and 5 D) 1,5 and 6 E) 1,2 and 5

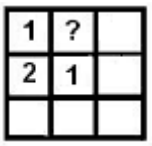
Solution: Since Zita walked from left to right, she had to pass by only one of the numbers in each group of two elements ({1,2}; {3,4};{5,6}).
Answer: C ({1,**2**}; {**3**,4};{**5**,6}).

4. What is the value of **2007 ÷ (2 + 0 + 0 + 7) – 2 × 0 × 0 × 7**?

A) 0 B) 9 C) 204 D) 223 E) 2007

Solution: It is important to follow the Order of Operations Rules. The expression simplifies to 2007/9-0=223.
Answer: D

5. The square in the figure is a *mini-sudoku*: the numbers 1, 2, and 3 must be written in the cells so that each of them appears in each row and in each column. Harry started to fill in the square. In how many ways can he complete the task?



A) 1 B) 2 C) 3 D) 4 E) 5

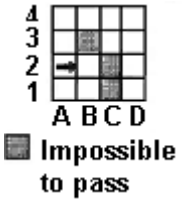
Solution: The number 3 must complete the second row as well as the first column, because the other two digits are already written there. It is not allowed to have the same digit written more than once in a row/column. Hence, in the second column, the number 3 can only be in the first row (where the "?" sign is). There is only one way to complete the task.
Answer: A.

6. Basil, who is older than Peter by one year minus one day, was born on January 1, 2002. What is the date of Peter's birthday?

A) January 2, 2003 B) January 2, 2001 C) December 31, 2000
D) December 31, 2002 E) December 31, 2003

Solution: Basil is *older*, so Peter was born *after* Basil (in time). We need to add to Basil's birthdate 1 year (so we get January 1, 2003) and then to take off a day. Peter was born on December 31, 2002.
Answer: D

7. A robot starts walking on the table from square A2 in the direction of the arrow, as shown on the picture. It always goes forward. If it reaches a barrier, it always turns right. The robot will stop if it he cannot go forward after turning right. On which square will it stop?

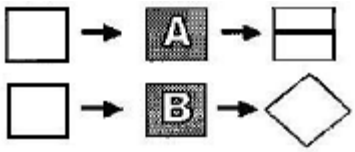


A) B2 B) A1 C) C3 D) D1 E) It will never stop

Solution: By the rule given, the robot will follow the route: A2-B2-B1-A1-A2-A3-A4-B4-BC4-D4-D3-D2-D1 and here, it will stop since it can't turn to the right.
Answer: D

8. The Carpenter's shop has two machines A and B. A is a "printing machine" and B is a "turning machine". What is the right sequence to obtain Figure 1 starting from Figure 2?

Figure 1 ***Figure 2***



A) BBA B) ABB C) BAB D) BA E) BABBB

Solution: Figure 2 must be marked and then, rotated on 90°. Machine A does the marking, while Machine B rotates on 45°.
Answer: B.

Part B: Each correct answer is worth 4 points.

9. A square is constructed on a square grid. Hanna coloured all the small squares placed on the diagonals of this square. What is the size of the side of the big square if the number of the coloured small squares is 9?

A) 3×3 B) 4×4 C) 5×5 D) 8×8 E) 9×9

Solution: There are 9 coloured squares on the two diagonals. However, there is one square, in the centre that belongs to both diagonals, thus, in fact there are 10 squares on both diagonals, or 5 squares for each. The number of squares on the diagonal is equal to the number of squares on its side, so the size of the square is 5x5.
Answer: C

10. Ana, Bianka, Clara, and Diana each play different sport: karate, soccer, volleyball, or judo. Ana does not like sports played with balls. The judo player Bianka often attends a soccer match to watch her friend play. Which of the following statements could be true:

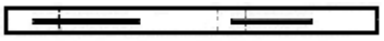
A) Ana plays volleyball B) Bianka plays soccer
C) Clara plays volleyball D) Diana plays karate E) Ana plays judo

Solution:

	Karate	*Soccer*	*Volleyball*	*Judo*
Ana	+	-	-	-
Bianka	-	-	-	+
Clara	-			-
Diana	-			-

Using the information given we can easily complete the above YES-NO table. Remember that we cannot have more than one sign + (or YES) on each row or column. According to the table, the only possible statement is C, "Clara plays volleyball".
Answer: C

11. Kelly has a paper ribbon 27 cm long. She divides it into four rectangles of different sizes and drew two segments such that every segment connected the centres of two adjacent rectangles (see the picture). What is the sum of the lengths of the two segments?



A) 12 cm B) 13.5 cm C) 14 cm D) 14.5 cm E) The answer depends on the division

Solution: Since the lines connect centres of rectangles, the length of each of them is equal to the sum of the halves of two of the rectangles. Since the four rectangles add up to the initial figure, the total length of the lines is equal to the half the length of ribbon, e.g., 27/2=13.5 cm.
Answer: B

12. Two 9 cm × 9 cm squares overlap to form a 9 cm × 13 cm rectangle, as shown. What is the area of the region where the two squares overlap?

A) 36 cm^2 B) 45 cm^2 C) 54 cm^2 D) 63 cm^2 E) 72 cm^2

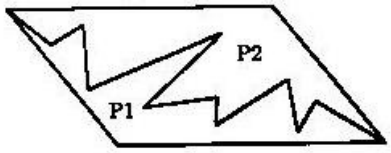
Solution: If the squares did not overlap, the length of the resulting rectangle would have been 18 cm. Since its length is 13 cm, therefore, the overlapping rectangle is 5 cm long. The other dimension of this rectangle is the same as the width of the original rectangles, 9 cm. Hence, the area is $5\times9=45$ cm^2.
Answer: B

13. There were 60 birds on three trees. At some moment 6 birds flew away from the first tree, 8 birds flew away from the second tree, and 4 birds flew away from the third tree. After that, it turned out that the number of birds on each tree was the same. How many birds were there on the second tree in the beginning?

A) 26 B) 24 C) 22 D) 21 E) 20

Solution: Solution: At the end, the total number of birds on the three trees was 60-6-8-4=42. There were the same number of birds on each tree, hence, there were 42/3=14 birds on each tree. Working backwards, we obtain
14+6=20 birds on the first tree;
14+8=22 birds on the second tree;
14+4=18 birds on the third tree.
Answer: C.

A parallelogram is divided into two parts, P1 and P2, as shown on the picture. Which of the following statements is surely true:



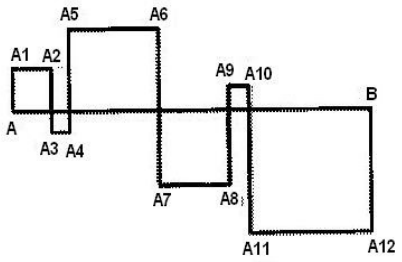
A) P2 has a greater perimeter than P1 B) P2 has a smaller perimeter than P1
C) P2 has a smaller area than P1 D) P1 and P2 have the same perimeter
E) P1 and P2 have the same area

Solution: The original parallelogram has two pairs of equal opposite sides. One side of each pair is included in the perimeters of P1 and P2. The dividing broken line is included in both perimeters, so it adds equal length to both of them. Hence, P1 and P2 have the same perimeters.
Answer: D.

14. What is the 2007th letter in the sequence KANGAROOKANGAROOKANG… ?

A) K B) A C) N D) R E) O
Solution: The sequence consists of repetitions of the 8-letter word KANGAROO. Since $2007=250\times8+7$, to write a sequence of 2007 letters we need to repeat the word 250 times and to include only 7 letters of the 251-th word. So, in the 2007th position, there will be the letter O.
Answer: E.

15. The squares on the figure are formed by intersecting the segment AB by the broken line $AA_1A_2...A_{12}B$. The length of AB is 24 cm. What is the length of the broken line $AA_1A_2...A_{12}B$?



A) 48 cm B) 72 cm C) 96 cm D) 56 cm E) 106 cm

Solution: The broken line consists of three sides from each of the squares, so its length will be ¾ of the total sum of their perimeters. On the other hand, the segment AB consists of one side of each square, so the length of AB is ¼ of the total sum of their perimeters. Hence, the length of the broken line is 3(AB)=72cm.
Answer: B

Part C: Each correct answer is worth 5 points.

16. Harry Potter let an owl out at 7:30 a.m. to deliver an important message to his friend Ron. The owl delivered the envelope at 9:10 a.m. An owl flies 4 km in 10 minutes. What was the distance between Harry and Ron?

A) 14 km B) 20 km C) 40 km D) 56 km E) 64 km

Solution: From 7:30 am to 9:10 am, it took one hour and 40 minutes (that is, 100 minutes) to get the message. The owl flies 4km in 10 minutes, so it would have flown (100/10)×4=40 km in total.
Answer: C

17. Bill thought of an integer number. Nick multiplied this number either by 5 or by 6. John added either 5 or 6 to Nick's result. Last, Andrew subtracted either 5 or 6 from John's result. The final result obtained was 73. What was Bill's number?

A) 10 B) 11 C) 12 D) 14 E) 15

Solution:

John	*Nick*	*Change from the last two operations*	*Result before the last two operations*
Added 5	*Subtracted 5*	*0*	*73-0=73*
	Subtracted 6	*-1*	*73+1=74*
Added 6	*Subtracted 5*	*+1*	*73-1=72*
	Subtracted 6	*0*	*73-0=73*

So, the possible results after Nick's multiplication are either 73, or 74, or 72. However, this result must be a multiple of either 5 or 6. The only possible result, thus, is 72. After reversing Nick's operation, we get 72/6=12. The original number of Bill was 12.
Answer: C

18. A cube with a side 1 m long has been cut into cubes of a side 1 dm each. All small cubes have been put one on top of the other, to form a vertical structure. What is the height of this structure?

A) 100 m B) 1 km C) 10 km D) 1000 km E) 10 m

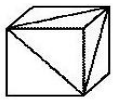
Solution: There are 10 dm in each meter. Therefore, the original cube is cut into 10×10×10=1000 cubes of a side 1 dm. When arranging them in a linear vertical structure, the height will be 1000dm=100m.
Answer: A

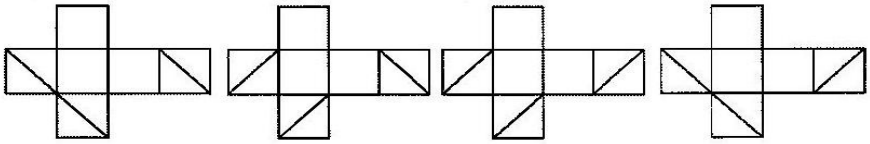
19. Agnes is 10 years old. Her mother Lisa is 4 times as old. How old will Lisa be when Agnes is twice as old as she is now?

A) 40 years old B) 50 years old C) 60 years old D) 70 years old E) 80 years old

Solution: Agnes is 10, so Lisa is 40. Agnes will be twice as old in 10 years. Then, Lisa will be 40+10=50 years old.
Answer: B

20. Three diagonals are constructed on three adjacent faces of a cube, as shown on the figure. Which of the following nets is that of the given cube?

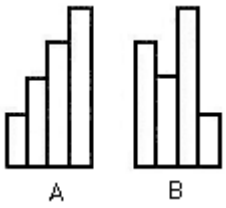




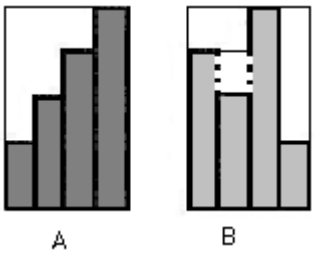
A) B) C) D)
E) None of the above

Solution: From the picture of the cube, it is clear that the three diagonals form a triangle, that is, every two of them share a vertex. So, the nets in B and C should be excluded. Imagine the nets in A and D folded into a cube. The rightmost upper vertex must meet the leftmost upper vertex. In D, this will be the common vertex of the two diagonals along the 4-squared piece of the net. In A, these diagonals will not share a vertex.
Answer: D.

21. Four paper ribbons of uniform width of 10 cm have been arranged to form figure A (see the diagram). Each of the ribbons is 25 cm longer than the previous one. The same ribbons have been rearranged to form figure B. What is the difference between the perimeters of figure A and figure B?



A) 0 cm B) 20 cm C) 40 cm D) 50 cm E) 25 cm



Solution: The perimeter of the original figure A is equal to the perimeter of the rectangle outscribed around it (as seen on the first figure on the left). To see this, just imagine where on the outer perimeter the segments from the inside of the rectangle will be projected if we decide to "unfold" the figure.
For the figure B, a similar reasoning will show that the perimeter of the original (grey) figure (except the two segments in dashed lines) will be projected on the outer perimeter, thus, it will be equal to the perimeter of the figure A, plus the two extra segments, the ones in dashed lines. Their length is 25 cm each. Therefore, the difference between the perimeters of A and B is 50 cm.
Answer: D

22. Anna wrote a 2-digit number. Ben created a 4-digit number by coping Anna's number twice. Then Anna divided Ben's number by her number. What was the result she got?
A) 100 B) 101 C) 1000 D) 1001 E) 10

Solution: Let Anna's number has *a* tens and *b* units, i.e., it is written by the digits *a, b,* in this order. Let us denote this number by $\overline{ab}$. Therefore, Ben's number will be written by the sequence of digits *a, b, a, b (*we will denote it by $\overline{abab}$*).* We can represent the number $\overline{abab}$ as follows: $\overline{abab} = 100 \times \overline{ab} + \overline{ab} = 100 \times \overline{ab} + 1 \times \overline{ab} = 101 \times \overline{ab}$. In the last step, we used the distributive property of multiplication. Therefore, when Ben's number $\overline{abab}$ is divided by Anna's number $\overline{ab}$ the result will be 101.

Answer: B.

23. Five integer numbers are written around a circle in a way that no two or three adjacent numbers have a sum divisible by 3. How many of these five numbers are divisible by 3?

A) 0 B) 1 C) 2 D) 3 E) Impossible to determine

Solution: The condition that the sum of any two adjacent numbers is not divisible by 3 translates into the fact that the remainders of these numbers when divided by 3 cannot be a sequence of 1 and 2, and cannot be two zeroes. So, two adjacent numbers can only have remainders (0 ,1), (1, 0), (0, 2), (2, 0), (1, 1) or (2, 2). On the other hand, the sum of any three adjacent numbers is also not divisible by 3, therefore, the combinations (0, 0, 0), (1, 1, 1), (1, 0, 2), or (2, 0, 1) are not possible.

Let us assume that there is no multiple of 3 among the five numbers. It follows that all five remainders are either 1 or 2, and evidently, they cannot be arranged to comply with the above restrictions.

Let us assume that there is only one multiple of 3. Then, the only possibilities for the numbers adjacent to it, are either 1 and 1 or 2 and 2. Without loss of generality, let us assume these numbers are 1 and 1. So, we already have a sequence of 1, 0, 1 and we must add two more numbers to the right of it. These numbers cannot be (1,1), or (2, 1) or (1, 2) or (2, 2) or (0, 2) or (2, 0). The possibilities are (1, 0) or (0, 1) only. Therefore, we will need two multiples of 3.

If we have three or more multiples of 3 among these five numbers, there always will be three of them adjacent to each other, which is not allowed.

Answer: C

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Bonus 1: *Each of the digits from 1 to 9 is used exactly once in the following multiplication example:*

$$\square\,\boxed{Y}\,\square \times \square\,\square = 7632$$

What digit is denoted by Y?

A) 1 *B) 4* *C) 5* *D) 8* *E) 9*

Note: Let us denote the digits in the empty positions as follows:

$$\boxed{X}\,\boxed{Y}\,\boxed{Z} \times \boxed{A}\,\boxed{B} = 7632$$

Solution:

The digits free to use are 1,4,5,8, 9. The prime factorization of 7632 is $2^4 \cdot 3^2 \cdot 53$. The factor 53 cannot be AB, since 3 is already used in the answer on the right. Hence, XYZ is a three-digit multiple of 53. Considering the prime factorization, XYZ=53×M, where M is one of the following numbers: 2, 3, 4, 6, 8, 9, 12, 16, or 18. (M≤18, since 53×18=954 is the greatest three-digit multiple of 53). Many of these numbers should be rejected because the number XYZ cannot include any of the digits 7, 6, 3, or 2. On the other hand, if M≥16, then the factors remaining from the prime factorization have a product that does not exceed 9, thus, cannot be a two-digit number AB. Taking into consideration all of these restrictions, the only possible choices for XYZ are

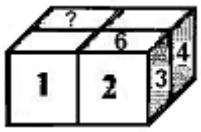
(1) 53×3=159, then AB=48. The multiplication becomes 159×48=7632, and it satisfies the requirements in the question, so, it is a possible solution.

(2) 53×8=424. Since there is a repetition of the digit 4, it is not a solution.

Finally, the value of the digit Y is 5.

Answer: C

Bonus 2: *In a regular die, the faces are numbered by the numbers 1 to 6 and the sum of the numbers on any two opposite faces is 7. Nick composed a rectangular prism* ***2×2×1*** *using four identical regular dice, with the numbers on any two touching faces of the dice being equal (see the figure). The numbers on some faces are shown. Which number must be written on the face denoted by the (**?**)?*



A) 5 *B) 6* *C) 2* *D) 3* *E) not enough information*

Solution:

Using the information that the numbers on any two opposite sides have a sum of 7, it is easy to see that the top and the bottom faces of the die that contains the "?" cannot be 6 and 1 or 3 and 4, thus, the "?" can only be 5 or 2. But taking into consideration the position of 2 and 3 relative to each other and to the rest of the digits, as well as the fact that all dice are identical, we obtain that "?" must be 5.

Answer: A.