International Contest-Game MATH KANGAROO
Canada, 2007

Grade 9 and 10

Part A: Each correct answer is worth 3 points.

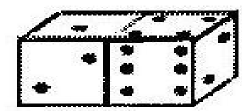
1. Anh, Ben and Chen have 30 balls altogether. If Ben gives 5 balls to Chen, Chen gives 4 balls to Anh and Anh gives 2 balls to Ben, then the three boys will have an equal number of balls. How many balls did Anh have at first?

A) 8 B) 9 C) 11 D) 13 E) 15

Solution: In the end, every boy has 30/3=10 balls. During the exchange, Anh gives two balls and receives 4 balls, so the number of his balls increased by 2. Hence, in the beginning, Anh had 10-2=8 balls.

Answer: A

2. A regular die has a total of 7 points on any two of its opposite faces. On the figure, two regular dice are placed, as shown. What is the sum of the points on all invisible faces of the dice?



A) 15 B) 12 C) 7 D) 27 E) another answer

Solution: On the right die, the numbers on the invisible faces are 1, 3, and 5. On the left die, the numbers on the invisible faces are 5, 6, 4, and 3. The total of all these numbers is 27. Alternatively, the total sum of all points on both dice is 7×3×2=42. The sum of the points we can see is 1+2+2+4+6=15, so the points on the invisible faces are 42-15=27.

Answer: D

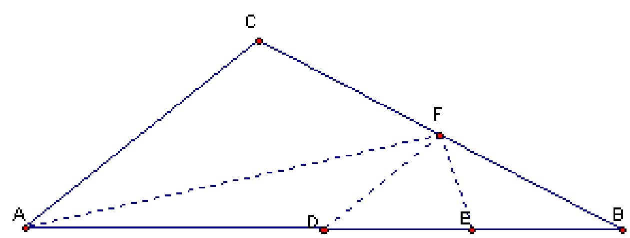
3. When announcing the results of a raffle, the moderator said: *"The winning tickets are those, which contain a number with at least 5 digits such that at most three of digits are greater than 2."* Subsequently, the speaker drew tickets with numbers 1022, 22222, 102334, 213343, 3042531. How many of them were winning ones?

A) 1 B) 2 C) 3 D) 4 E) 5

Solution: The winning numbers are 22222 and 102334. 1022 has less than 5 digits, while 213343 and 3042531 have more than three digits greater than 2.

Answer: B

4. In the triangle ABC, D is the midpoint of AB, E is the midpoint of DB, F is the midpoint of BC. If the area of triangle ABC is 96, what is the area of triangle AEF?



A) 16 B) 24 C) 32 D) 36 E) 48

Solution: Denote by [ABC] the area of the triangle ABC. Recall that in a triangle, each median splits it in two triangles with the same area. In triangle ABC, AF is a median. Therefore, [ABF]=1/2[ABC]. In triangle ABF, DF is a median. Hence, [DBF]=1/2[ABF]=1/4[ABC]. In triangle DBF, FE is a median, so, [BEF]=1/2[DBF]=1/8[ABC]. As a result, [AEF]=[ABF]-[BEF]=1/2[ABC]-1/8[ABC]=3/8[ABC]=3/8(96)=36.

Answer: D

5. Frida has 2007 marbles kept in three bags, A, B, and C. Each bag contains the same number of marbles. If Frida moves 2/3 of the marbles in bag A to bag C, what will be the ratio between the number of marbles in bag A and bag C?

A) 1:2 B) 1:3 C) 2:3 D) 1:5 E) 3:2

Solution: In the beginning, each bag contains 1/3 (or 3/9) of all marbles. When Frida moves 2/3 of the marbles in bag A to bag C, the marbles left in A are 1/9 of all marbles, while the marbles in bag C increase from 3/9 to 5/9 of all marbles. The ratio A:C=1:5.

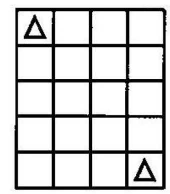
Answer: D

6. An international organisation has 32 members. It is predicted that the organisation will increase the number of its members by 50% each year. How many members will the organisation have in three years?

A) 182 B) 128 C) 108 D) 96 E) 80

Solution: The increase by 50% every year is equivalent to multiplying the number of members in the beginning of the year by a factor of 1.5, in order to calculate the number of members in the end of the year. Therefore, in three years the organization will have
1.5(1.5(1.5(32)))=1.5(1.5(48))=1.5(72)=108 members.

Answer: C.



7. In one move, the King can go to any adjacent square, along a row, column, or diagonal. How many routes with the minimum number of moves are there for the king to travel from the top left square to the bottom right square on the grid?

A) 1 B) 4 C) 7 D) 20 E) 35

Solution: The minimum number of steps is 4 (three along a diagonal and one down). The variety of routes depends on whether the step down will be taken on the first, second, third, or fourth move, so, there are 4 different routes with a minimum number of steps required.

Answer: B.

8. What is the least possible value of the expression 2007 – KAN – GA – ROO, if it is given that each letter represents a digit (different letters represent different digits and the same letters – equal digits)?

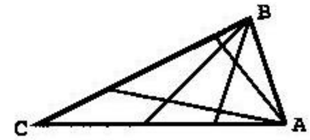
A) 100 B) 110 C) 112 D) 119 E) 129

Solution: In order to minimize the difference, we must maximize the numbers that are subtracted from 2007. The greatest possible digits 9 and 8 must be used for K and R, 7 and 6 must be used for the digits O and A, as they appear twice and therefore, have a greater influence on the numbers compared to the digits G and N that only appear once. Consequently, G=5, N=4. As a result, the expression becomes 2007-974-57-866=110. This is the minimum possible value. (Switching the positions of 9 and 8 and/or 7 and 6 will lead to the same minimal result for the difference).

Answer: B

<u>Part B: Each correct answer is worth 4 points.</u>

9. On the diagram, the triangle ABC is divided into nine non-overlapping sections by drawing two lines from each of the vertices A and B. How many non-overlapping sections will there be if four lines are drawn from each of these vertices?



A) 16 B) 25 C) 30 D) 42 E) 49

Solution: Drawing five lines from A divides the triangle into 5 sections. Then, each of the lines from B will divide each of these five sections into five parts. Hence, the total number of sections obtained this way is 5×5=25.
The result can be generalized: when *n* lines are constructed from each of the two vertices, the number of sections obtained is $(n+1)\times(n+1)$.

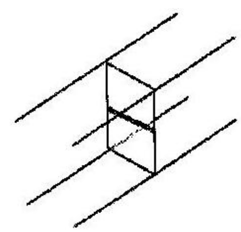
Answer: B

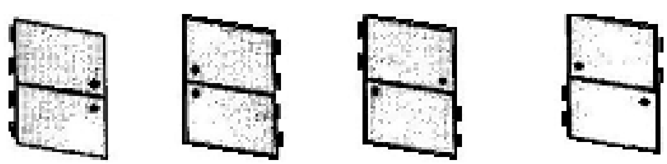
10. An island is inhabited by liars and nobles (the liars always tell lies and the nobles always tell the truth). One day, 12 islanders, among them both liars and nobles, gathered together and issued a few statements. Two people said: “Exactly two people among us twelve are liars”. Another four people said: “Exactly four people among us twelve are liars”. The remaining six people said: “Exactly six people among us twelve are liars”. How many liars were there?

A) 10 B) 8 C) 6 D) 4 E) 2

Solution: It is possible that all the three statements are false, thus, all 12 people are liars, but, from the given, it is assumed that there is at least one noble (besides, 12 is not a possible answer). If any of the first two people is a noble, it means that there are exactly two liars in the group. But it implies that both the second and the third statement are false, so all 10 remaining people are liars, which contradicts to the assumption that the first statement is true. So, both people issued the first statement are liars. Similarly, all four people issuing the second statement are liars.
If any of the last group of people is noble, it follows that there are exactly six liars, which will be possible if the first two groups of people are liars and the last group of six people are all nobles.
Answer: C

11. A hallway is sagged on the right side. As a consequence, the profile is no longer a rectangle but a parallelogram. There is a door halfway through the hallway. The door has two sections, which can be opened separately. Where should they put the hinges?

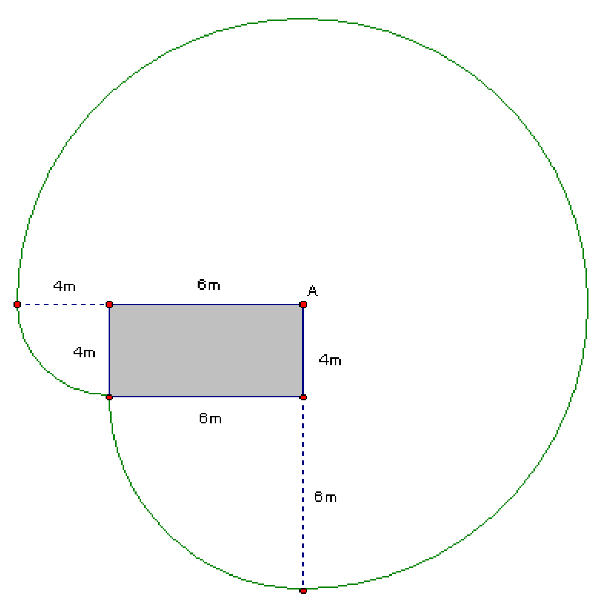




A) both left B) both right C) above left, below right D) below left, above right E) The door can never open properly

Solution: Because of the cross-section that is a parallelogram, the top edge should be such that the corner on the side of the hinges is higher than the other top corner. On the other hand, the bottom edge should allow for the corner on the side of the hinges to be lower than the other corner. Otherwise, the door will get stuck on the ceiling or the floor of the hallway.
Answer: C

12. A 10 m rope is fastened to one of the corners of a house, which has the form of a rectangle, 6 m long and 4 m wide. A dog is fastened to the rope. What is the perimeter of the region that the dog can access?



A) 20π B) 22π C) 40π D) 88π E) 100π

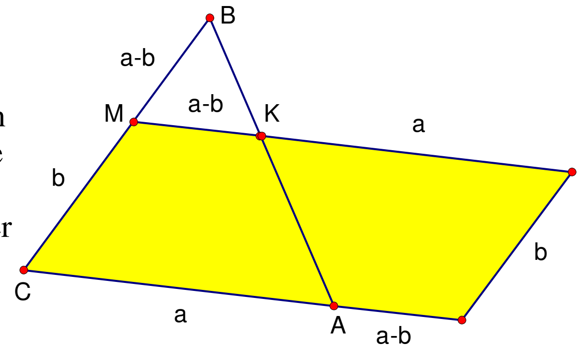
Solution: As seen from the diagram, the region accessible for the dog is composed of three quarters of the circle with a radius of 10 m, and a quarter of each of the two circles with a radius of 4 m and 6 m.
Applying the formula for the circumference of a circle with a radius of R, $C = 2\pi R$, we can calculate the perimeter of this region, in meters:

$$Perimeter = \frac{3}{4}(20\pi) + \frac{1}{4}(12\pi) + \frac{1}{4}(8\pi) =$$

$$= 15\pi + 3\pi + 2\pi = 20\pi$$

Answer: A

13. A trapezoid is formed by removing a corner from an equilateral triangle. Two copies of this trapezoid are placed side by side to form a parallelogram. The perimeter of the parallelogram exceeds the perimeter of the original triangle by 10 cm. What is the perimeter of the original equilateral triangle?



A) 10 cm B) 30 cm C) 45 cm D) 60 cm E) more information needed

Solution: Denote the side of the triangle by *a* and the length of the segment CM by *b* (see the diagram). Then, $P(triangle) = 3a,$ while $P(paralleogram) = a + (a - b) + b + a + (a - b) + b = 4a.$ Therefore, $P(paralleogram) - P(triangle) = 4a - 3a = a.$ It follows that the side of the triangle is 10cm, hence, the perimeter is 30 cm.
Answer: B.

14. A sequence of letters KANGAROOKANGAROO…KANGAROO contains 20 words KANGAROO. First, all letters in the odd positions of the sequence were erased. Then, in the sequence obtained, once again all the letters in the odd places were erased, and so on. At the very end, only one letter remained. What is this letter?

A) K B) A C) N D) G E) O

Solution: Completing the described operation on the given sequence, after the first step we obtain a sequence AGROAGRO…., after the second step – the sequence GOGOGO…, and after the third step, the sequence OOO…. Evidently, the last letter remained will be O.
Answer: E.

15. What percent of the natural numbers from 1 to 10000 are perfect squares? (*Perfect square is a number that can be presented as a square of a natural number, for instance* $100 = 10^2$).

A) 1% B) 1.5% C) 2% D) 2.5% E) 5%

Solution: $1=1^2$. $10000=100^2$. Since the squares of all numbers between 1 and 100 must be some numbers in the interval between 1^2and100^2, out of the natural numbers up to 10000 exactly 100 numbers are perfect squares, which is 100/10000=1/100=1%.
Answer: A

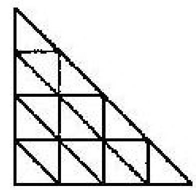
16. The number 8^8 is obtained from the number 4^4 by raising this number to the power of **n.** What is the number **n**?

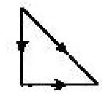
A) 2 B) 3 C) 4 D) 8 E) 16

Solution: From the exponent rules, $8^8 = (2^3)^8 = 2^{24}$; $(4^4)^n = ((2^2)^4)^n = (2)^{8n}$. So, n=3.
Answer: B.

Part C: Each correct answer is worth 5 points.

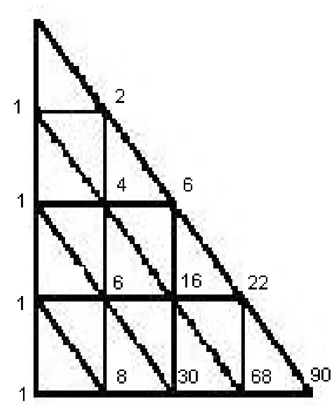
A map of a neighbourhood is shown on the figure. The neighbourhood has a form of a right-angled triangle. All streets are shown on the map. They divide the neighbourhood into blocks that also are right-angled triangles. Alex wants to drive from the top endpoint of the neighbourhood to the rightmost endpoint of the neighbourhood. The traffic regulations only allow going down (vertically), right (horizontally), or down by a "hypotenuse" of a block. From how many different routes can Alex choose?

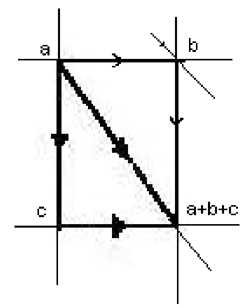




A) 16 B) 27 C) 64 D) 90 E) 111

Solution: The traffic regulations are such that each point on the map can only be reached either from the point directly to the right of it, or through the point directly above it, or from the point along the diagonal coming from the left up. Therefore, the number of different routes leading to this point is the sum of the number of different routes leading to any of the three points just before it (see the small fragment of the map on the right diagram).





Keeping track in consequence on the different routes that are available to each point on the map, we conclude that the rightmost bottom point can be reached in 90 different routes (see the map in the diagram: the numbers beside each intersection represents the number of different routes that lead to this intersection).

Answer: D.

17. The first digit of a 4 – digit number is equal to the number of zeroes in this number. The second digit of the number is equal to the number of digits 1, the third digit is equal to the number of the digits 2, and the fourth digit represents the number of the digits 3 in this number. How many numbers have this property?

A) 0 B) 2 C) 3 D) 4 E) 5

Solution: Denote by D1, D2, D3 and D4 the digits of the number. Since D1 is the number of zeros and we have a four-digit number, D1 has to be either 1, or 2, or 3.

Case 1: D1=3, so the number must contain three zeroes, but the number 3000 does not satisfy the requirements (the number of the digits 3 in D4 is not correct). So, there is no such number that starts with 3.

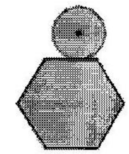
Case 2: D1=2. Then, we need two zeroes, but D3 cannot be 0, since we already used one digit 2, so the zeroes must be D2 and D4. The choice for D3 must be made from 1 or 2. It is easy to check that 2010 does not have the required property while 2020 has it.

Case 3: D1=1. Then, the number must contain one digit 0, which cannot be D2 (since we already used a digit 1). On the other hand, the digit D2 cannot be 1, because in this case there will be at least two digits 1 in the number and $2 \neq$D2. The digit D2 also cannot be 3, because in this case we need to place three digits 1 in the number, which is not possible (only one undetermined digit out of the four is left). So, the number must be 12_ _ , where one of the D3 and D4 is 0 and the other one is 1. The number 1210 is the only one that satisfies all requirements in this case.

In conclusion, there are two numbers that have the required property.

Answer: B

18. A coin with a diameter 1 cm rolls around the outside of a regular hexagon with a side length of 1 cm, as shown. What is the length (in cm) of the path traced out by the centre of the coin after one complete rotation around the hexagon?



A) $6 + \pi/2$ B) $6+\pi$ C) $12 +\pi$ D) $6+2\pi$ E) $12 +2\pi$

Solution: The centre of the coin traces a path that is composed by six segments parallel and equal to the six sides of the hexagon and six arcs at the vertices. Each arc corresponds to a sector of a circle with a radius of 0.5 cm and an angle at the centre equal to 60°. Therefore, the six arcs, together, form a complete circle. The length of the path equals $6\times1+2\pi(0.5)=6+\pi$.

Answer: B

19. Let A be the least natural number with the following property: 10×A is a perfect square (a second exponent of a natural number) and 6×A is a perfect cube (a third exponent of a natural number). How many positive divisors does the number A have?

A) 30 B) 40 C) 54 D) 72 E) 96

Solution: A number is a perfect square if and only if all prime factors in its prime factorization are taken to an even exponent. A number is a perfect cube if and only inf all prime factors in its prime factorizaton are taken to an exponent that is divisible by 3. Since 10A is a perfect square, A must have 2 and 5 (taken to odd exponents) as prime factors, i.e., $2^{2n+1}\cdot5^{2k+1}$ is part of the prime factorization of A. On the other hand, since 6A is a perfect cube, $2^{3m-1}\cdot3^{3r-1}$ is also part of the prime factorization of A. The least number that can be represented both in the form (2n+1) and (3m-1) for some integer numbers n and m is 5 (for n=2, m=2), so the least power of 2 in the prime factorization of A is 2^5. The power of 3 must have an exponent that is of the form (3r-1) for some r and is even (as A is a perfect square). The power of 5 must have an exponent that is odd and a multiple of 3. The least such number is 3. As A is the least number that satisfies the conditions in the question, $A=2^5\cdot3^2.5^3$. We need to calculate the number of positive divisors of A. Each divisor is a combination of 2s, 3s, and 5s, where the number of factors 2 can be 0, 1, 2, ,3, 4, or 5; the number of factors 3 can be 0, 1, or 2; and the number of factors 5 can be 0, 1, 2, or 3. Therefore, there are 6×3×4=72 different combinations of factors. Please note that the combination $2^0\cdot3^0.5^0=1$ (the least factor of A) and the combination $2^5\cdot3^2.5^3=A$ (the greatest factor of A).

Answer: D.

20. On a party, five friends are going to give each other gifts in such a way that everybody gives one gift and receives one gift (of course, no one should receive their own gift). In how many ways is this possible?

A) 5 B) 10 C) 44 D) 50 E) 120

Solution: Denote the friends by A, B, C, D and E. The set of possible distributions of gifts can be split into two non-overlaping subsets: (1) Among the 5 friends, there are two who exchanged their gifts (i.e. A gets the present of B and B gets the present of A). (2) Among the 5 friends, there is no such pair that exchanged their gifts. We will count the possibilities for each of the two subsets.

(1) Assume that two of the friends, say A and B, echanged their gifts. It follows that among the other three friends, there weren't a pair that also exchanged their gifts, because otherwise, the fifth person will have nobody to give his/her gift to, which is not allowed. For the three friends, C, D, and E, there are only two possibilities to exchange gifts: C gives to D, D gives to E, E gives to C or C gives to E, E gives to D, D gives to C. It follows that the number of the possibilities in this subset can be calculated by doubling the possibilities to choose two friends out of the five friends (which is equal to ${}_5^2C=\binom{5}{2}=\frac{5\times4}{1\times2}=10$).

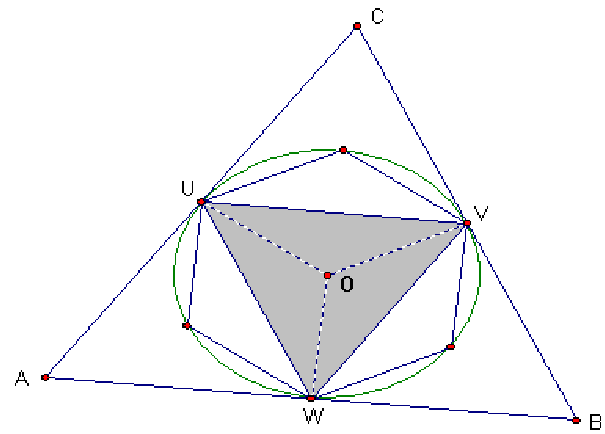
Therefore, there are 20 possible ways to exchange gifts in a way that two friends would exchange gifts between themselves.

(2) Assume that no two friends exchanged their own gifts. Without loss of generality, let A gives a gift to B. Then, B cannot give the gift to A, so, B gives the gift to somebody of C, D, or E. Assume, B gives the gift to C. Similarly, C must give to either D or E, assume, to D, and, finally, D gives to to E and E gives to A, hence, the exchange is only possible " around the circle", where the five friends are arranged in any order. Therefore, the number of possibilities in this subset is equal to the number of diferent arrangements of five elements around a circle. Note that two arrangements around a circle are different if and only if there is at least one element that has at least one neighbour that is different. To count the number of there arrangements, first, consider all possible arrangements of five elements in a row (there are 5!=120 such arrangements). Then, note that, in terms of the definition of different arrangements around the circle, there will be groups of arrangements that are not distinct, for instance, ABCDE is equivalent to BCDEA, CDEAB, DEABC, and EABCD. More specifically, for each permutation of the five elements, there will be four more that are equivalent to it. Therefore, the number of different arrangements of five elements around a circle is 120/5=24.

In conclusion, the number of possible ways the five friends can exchange gifts is 20+24=44.

Answer: C.

21. An equilateral triangle and a regular hexagon are inscribed in a circle, which itself is inscribed in a larger equilateral triangle (see the figure). S1 denotes the area of the greater triangle, S2 is the area of the smaller triangle, and S3 is the area of the hexagon. Which of the following equalities is true?



A) $S3 = \sqrt{S1 \times S2}$ B) $S3 = \dfrac{S1 + S2}{2}$ C) $S1 = S2 + S3$

D) $S3 = \sqrt{S1^2 \times S2^2}$ E) $S1 = S3 + 3 \times S2$

Solution: Recalling the properties of equilateral triangles, regular hexagons and circles incribed or outscribed around them, it is not difficult to prove that the centre of the circle, O, is the centre of all figures. The segment OU is perpendicular to the side AC of the greater triangle (properties of tangents to a circle). Therefore, OU is part of the height (and the median) of the equilateral triangle ABC, hence, U is the midpoint of the side AC. Similarly, V is the midpoint of BC and W is the midpoint of AB. From the side-split theorem, it follows that the side of triangle UVW is a half of the side of triangle ABC, therefore, the area of UVW is one quarter of the area of ABC. In terms of the notation introduced in the problem, this means that (*) S1=4(S2). On the other hand, the area of the hexagon is exactly twice the area of triangle UVW (looking at the diagram, it is easy to notice that the area of the hexagon can be seen as a composition of the areas of six congruent triangles, each of them representing one third of the area of the triangle UVW). So, it follows that (**) S3=2(S2). Using the results in (*) and (**), we calculate that

$$S1 \times S2 = 4 \cdot (S2)^2 = (2(S2))^2 = (S3)^2 \Rightarrow S3 = \sqrt{S1 \times S2}.$$

Answer: A.

22. It is 21:00 hours (military time). I am driving at 100 km/h. With this speed I have enough gas to pass 80 km. The nearest gas station is 100 km away. The amount of gas my car uses per 1 km is proportional to the velocity of the car. I want to reach the gas station as soon as possible. At what time, at the earliest, can I arrive at the gas station?

A) 22:12 B) 22:15 C) 22:20 D) 22:25 E) other answer

Solution: It is clear that the speed must be reduced in order to decrease the gas used per 1 km. We need to find the greatest speed, *v*, that the supply of gas allows in order to reach the closest gas station. Denote the amount of gas used per 1 km at 100 km/h by *a*, and the amount of gas used per 1 km at the new speed by *b*. Since the amount of gas per 1 km is proportional to the speed, it follows that $\frac{100}{a} = \frac{v}{b} \Leftrightarrow 100b = av$. On the other hand, the gas in the car is *80a*, and we want it to be just enough for travelling 100 km, so it must equal *100b.* So, *80a=100b,* and further, *80a=av,* which means that *v=80km/h.* At 80 km/h, it takes 100/80=1.25 h to travel 100 km. So, the gas station can be reached in 1h 15 min., at the earliest (i.e., at 22:15).

Answer: B.

23. There are three green, three red, three blue and three yellow cards in a box. Each of the three cards from each colour is numbered by a number 1, 2, or 3 (e.g. there is one green card with a number 1, one green card with a number 2, one green card with a number 3, etc.). We take randomly three cards from the box. Which of the following events is the most probable one (has the greatest probability)?

A) The three cards are of the same colour.
B) The three cards, regardless of their colours, have numbers 1, 2, and 3.
C) The three cards are of three different colours.
D) The three cards have the same number on them.
E) None of the above, the four previous events have the same probability.

Solution: Recall that the probability of any event is calculated as a ratio of the favourable outcomes and all possible outcomes. In all situations A-D, the number of favourable outcomes is the same, and equals the possible ways to choose three cards out of 12 cards. We calculate this number by ${}_{12}^{3}C = \frac{12 \cdot 11 \cdot 10}{1 \cdot 2 \cdot 3} = 220.$ The probability will be greatest when the number of favourable outcomes is the greatest.

Let us calculate the number of favourable outcomes for each of the events A to D.

A. It is possible to choose three cards of the same colour in only four ways, either all three green cards, or all three red cards, or all three blue cards, or all three yellow cards, so, the number of favourable outcomes for event A is 4.
B. Group the cards with the same number together; as a result, we form three groups of four cards in each of them. Choosing three cards that have number 1, 2, and 3, regardless their colour, is equivalent to choose one card from each group. There are four ways to choose one card from one group, and these choices are independent, so, the ways to choose the required three cards are 64 (4×4×4).
C. For a selection of three colours, we can select three cards –one from each of these colours – in 27 ways (for each of the cards, we have 3 possible ways to be chosen, and the choices are independent, so the total number of possibilities is 3×3×3=27). Three colours can be chosen from the four colours in 4 possible ways (it is equivalent to choose a colour to be

excluded from the selection, which, definitely, can be done in 4 different ways). Therefore, the number of favourable outcomes for this event is 4 ×27=108.

D. Considering the three groups of four cards defined for the event B, choosing three cards of the same number is equivalent to choose three cards of any of these groups. From each group, three cards can be chosen in 4 different ways. The total number of favourable outcomes is equal to the sum of possible choices from each of the four groups, that is, 4+4+4=12.

Since the even C has the greatest number of favourable outcomes, this event has the greatest probability.

Answer: C.

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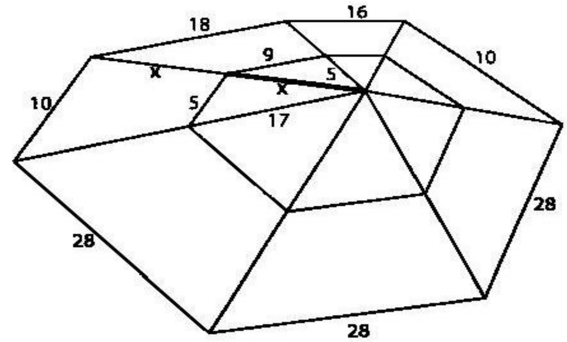


Bonus 1: *A mathematically skilled spider spins a web and some of the strings have lengths as shown in the picture. If x is an integer, determine the value of x.*

A) 11 *B)*13 *C)* 15 *D)* 17 *E) 19*

Solution: The key to solving this problem is the triangle inequality that states that three segments, a, b, and c, can form a triangle if and only if each of them is greater than the difference of the other two and smaller than the sum of the other two segments.

Assuming (without loss of generality) $a \geq b \geq c$, the inequalities that represent this statement are: *b+c>a>b-c; a+c>b>a-c; a+b>c>a-b.*

In the net, there are two triangles with a side x. Applying the triangle inequality for each of them, it follows that x<5+9=14 and x>17-5=12. The only integer number that satisfies both conditions is 13, therefore, x=13.

Answer: B.

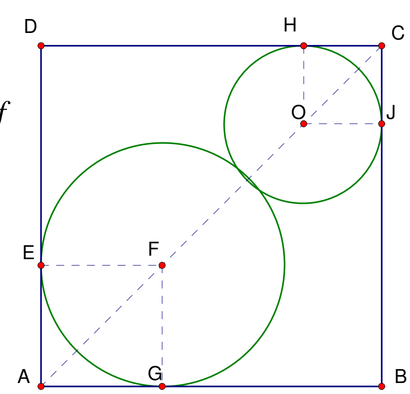
Bonus 2: *There are several necklaces in a safe deposit. All the necklaces have the same number of diamonds (at least two diamonds in each necklace). If the number of diamonds in the safe deposit would be known, then the number of the necklaces would also be certainly known. There are more than 200 and less than 300 diamonds in the safe. How many necklaces are there?*

A) 16 B) 17 C) 19 D) 25 E) other answer

Solution: The number of all diamonds in the safe is equal to the number of necklaces multiplied by the number of diamonds in each necklace; hence, it is a multiple of both these numbers. It is given that if the total of all diamonds in the safe is known then the number of necklaces will be certainly known. This implies that the total number of diamonds has only one factor except 1 and itself, therefore, it is a second power of a prime number. Otherwise, there will be more than one possibility for the number of necklaces. The only such number in the range 200-300 is $289=17^2$. Hence, there are 17 necklaces and 17 diamonds in each of them.

Answer: B.

Bonus 3: *Two circles have their centres on one of the diagonals of a square. They touch each other and the sides of the square, as shown. The square has a side length of 1 cm. What is the sum of the lengths of the two radii of the circles, in centimetres?*



A) $\frac{1}{2}$ *B)* $\frac{1}{\sqrt{2}}$ *C)* $\sqrt{2}-1$ *D)* $2-\sqrt{2}$ *E) It depends on the ratio of the two radii*

Solution: Denote the centres of the circles by O and F and their radii by r and R, respectively. The circles are tangent to the sides of the square, so, the perpendiculars from the centres to the sides have a length equal to the radii. Hence, AGFE and OJCH are squares with a side of R and r, respectively. Consecutively, AF=R√2, and OC=r√2, as diagonals of the squares. Now, AC=AF+FO+OC= R√2+(R+r)+ r√2=(1+√2)(R+r).
On the other hand, AC is the diagonal of the original square, so its length is √2. The sum of the lengths of the two radii can be calculated from the equation: (1+√2)(R+r)= √2. In conclusion, $R+r=\frac{\sqrt{2}}{1+\sqrt{2}}=\sqrt{2}\left(\sqrt{2}-1\right)=2-\sqrt{2}$.

Answer: D.